Calculation of fourside supported Slab

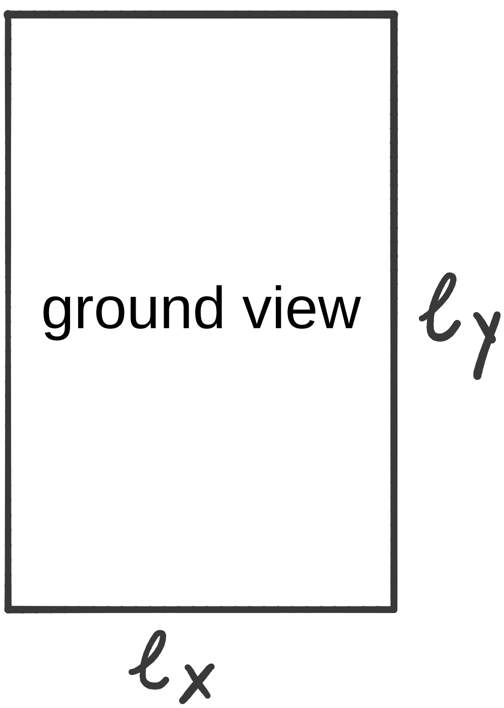
What do we need for the calculation?

1. Dimensions
2. Materials
3. Loads

What steps do we take for the calculation?

1. Characteristic Loads -> Design Loads
2. Calculation of Deformation
3. Design of Reinforcement

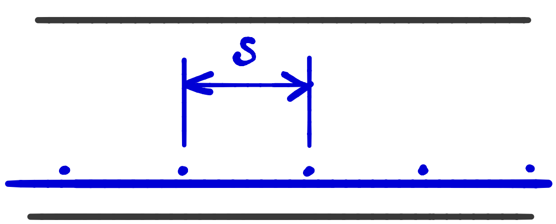
1. Dimensions



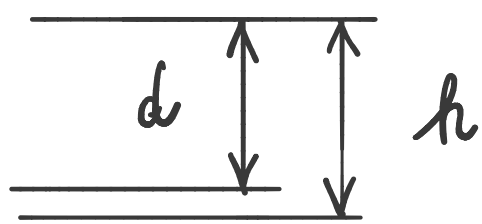
Supporting Spans
(e.g. middles of walls)

x-direction = short span

4 side support, openings
< 1.5 m, < 1/4 of span



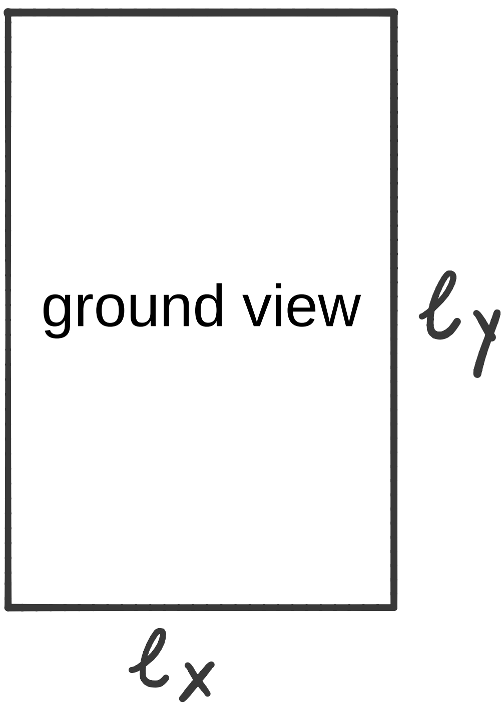
cut



h = height of slab

d = statical height

1. Dimensions

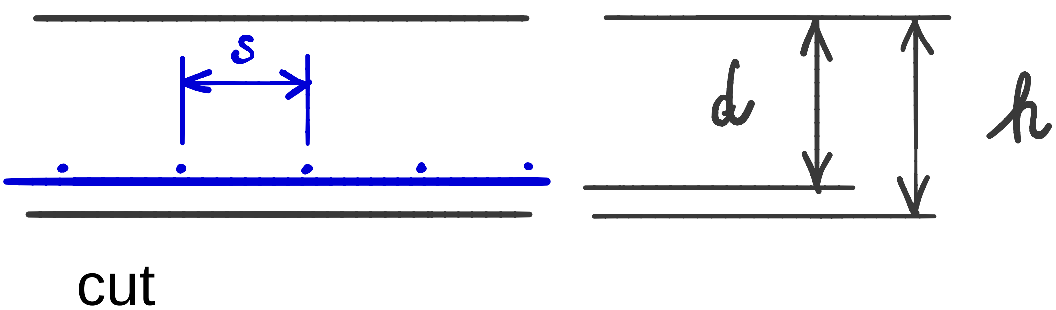


example:

$\ell_x = 5.0\,m$

$\ell_y = 7.5\,m$

$$\frac{\ell_y}{\ell_x} = \frac{7{,}5}{5{,}0} = 1{,}5 \text{ span ratio}$$



cut

$h = 160\,mm$

$d = 125\,mm$

2. Materials

Concrete:

C20/30 PC275

C25/35 PC300

C30/37 PC325

E-Modulus

30'000 .... 35'000

$E \approx 32'500 \frac{N}{mm^2}$

with professional use of average concrete,
for slabs concrete will never be critical part of design

2. Materials

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C20/30 PC275
C25/35 PC300
C30/37 PC325

E-Modulus

$E \approx 32'500 \frac{N}{mm^2}$

with professional use of average concrete,
for slabs concrete will never be critical part of design

Steel:

S500
B500B
B500

-> f_y = 460 N/mm2 (yield)

$f_{dy} = f_y/1.05$

$f_{yd} = 435 \frac{N}{mm^2}$

check your local steel quality

3. Loads

Concept of Design Loads (d)

In static calculation always check two basic topics:

a) Serviceability (d,ser)

-> Deformations
-> Cracks/Aesthetics
-> Vibrations, etc.

b) Structural Safety (d)

-> Calculation of required Reinforcement
-> Check of tensile/compession Pressures
-> Check of Settling

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Concept of Design Loads (d)

In static calculation always check two basic topics:

a) Serviceability (d,ser)

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-> Cracks/Aesthetics
-> Vibrations, etc.

↳ Deformation of Slab

b) Structural Safety (d)

-> Calculation of required Reinforcement
-> Check of tensile/compession Pressures
-> Check of Settling

↳ Required Reinforcement

3. Loads

3a. Characteristic Loads

-> real and effective loads without any factors

Self Weight g

Dead Load a

Life Load n

3. Loads

3a. Characteristic Loads

-> real and effective loads without any factors

Self Weight g

$$g = \underset{h\ \text{slab}}{0{,}16} \cdot \underset{\text{concrete}}{25} = 4{,}0\ \text{kN/m}^2$$

Dead Load a

$$a = \underset{\text{split/sand}}{0{,}04 \cdot 20} + \underset{\text{garden slabs}}{0{,}04 \cdot 24} = 1{,}76\ \text{kN/m}^2$$

Life Load n

$$n = 3{,}0\ \text{kN/m}^2$$

according swiss codes for a terrace

3. Loads

3b. Design Loads for Deformation

-> Security factors according codes (partly reduction due to probability of appearance)

Self Weight g -> $g_{d,ser}$: Factor 1,0

Dead Load a -> $g_{d,ser}$: Factor 1,0

Life Load n -> $g_{d,ser}$: Factor 0,3

3. Loads

3b. Design Loads for Deformation

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Self Weight g -> $g_{d,ser}$: Factor 1,0

Dead Load a -> $g_{d,ser}$: Factor 1,0

Life Load n -> $g_{d,ser}$: Factor 0,3

$$\underline{p_{d,ser}} = 1{,}0 \cdot 4{,}0 + 1{,}0 \cdot 1{,}76 + 0{,}3 \cdot 3{,}0$$

$$= 6{,}66 \approx \underline{6{,}7\ kN/m^2}$$

3. Loads

3c. Design Loads for Structural Safety

-> Security factors according codes (Life Load acting as main impact)

Self Weight g -> $g_{d,ser}$: Factor 1,35

Dead Load a -> $g_{d,ser}$: Factor 1,35

Life Load n -> $g_{d,ser}$: Factor 1,5

3. Loads

3c. Design Loads for Structural Safety

-> Security factors according codes (Life Load acting as main impact)

Self Weight g -> $g_{d,ser}$: Factor 1,35

Dead Load a -> $g_{d,ser}$: Factor 1,35

Life Load n -> $g_{d,ser}$: Factor 1,5

$$\underline{p_d} = 1{,}35 \cdot 4{,}0 + 1{,}35 \cdot 1{,}76 + 1{,}5 \cdot 3{,}0$$

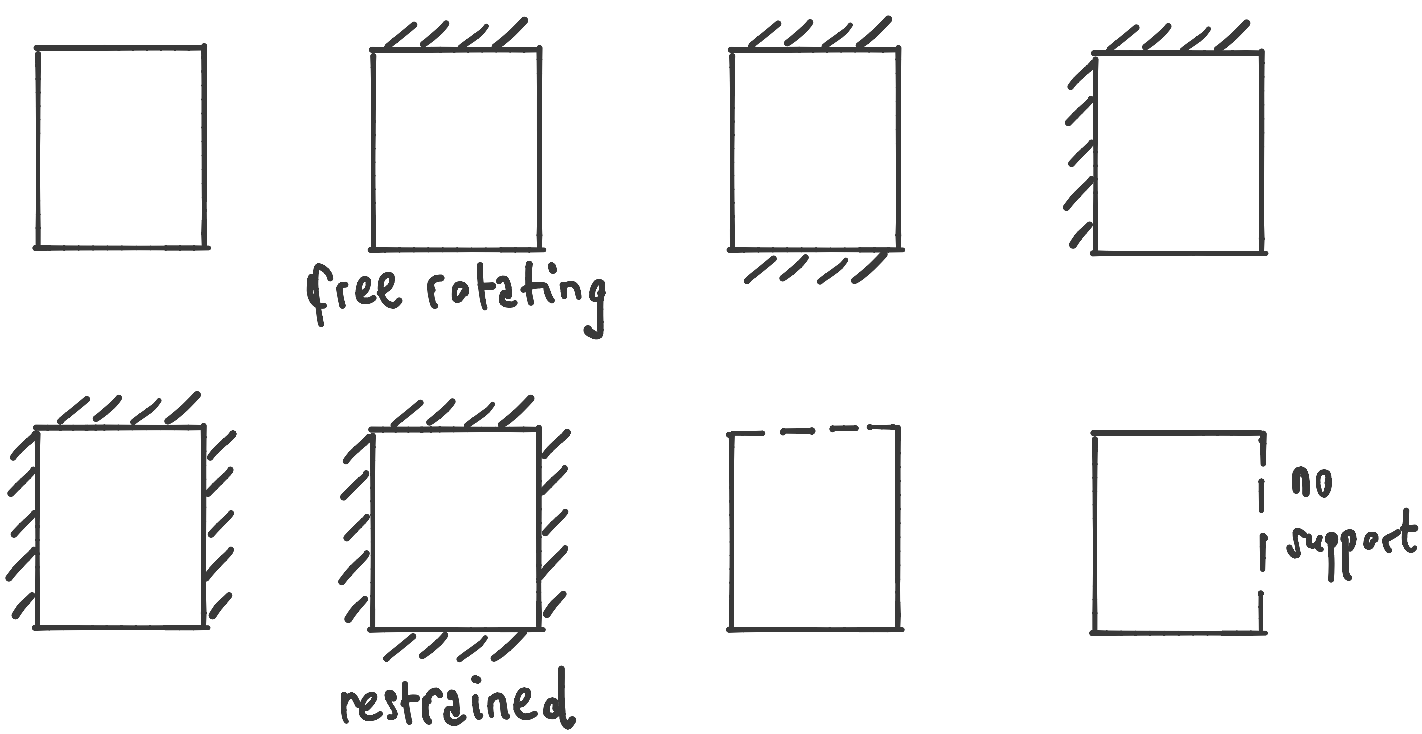
$$= 12{,}28 \approx \underline{12{,}3\ kN/m^2}$$

4. Czerny Table(s)

Austrian engineer who put the values for two directional bearing slabs in tables

Tables have a range from 1:2 to 2:1 for span ratios lx/ly

There are more tha 20 different tables for all kind of support cases and load distributions:



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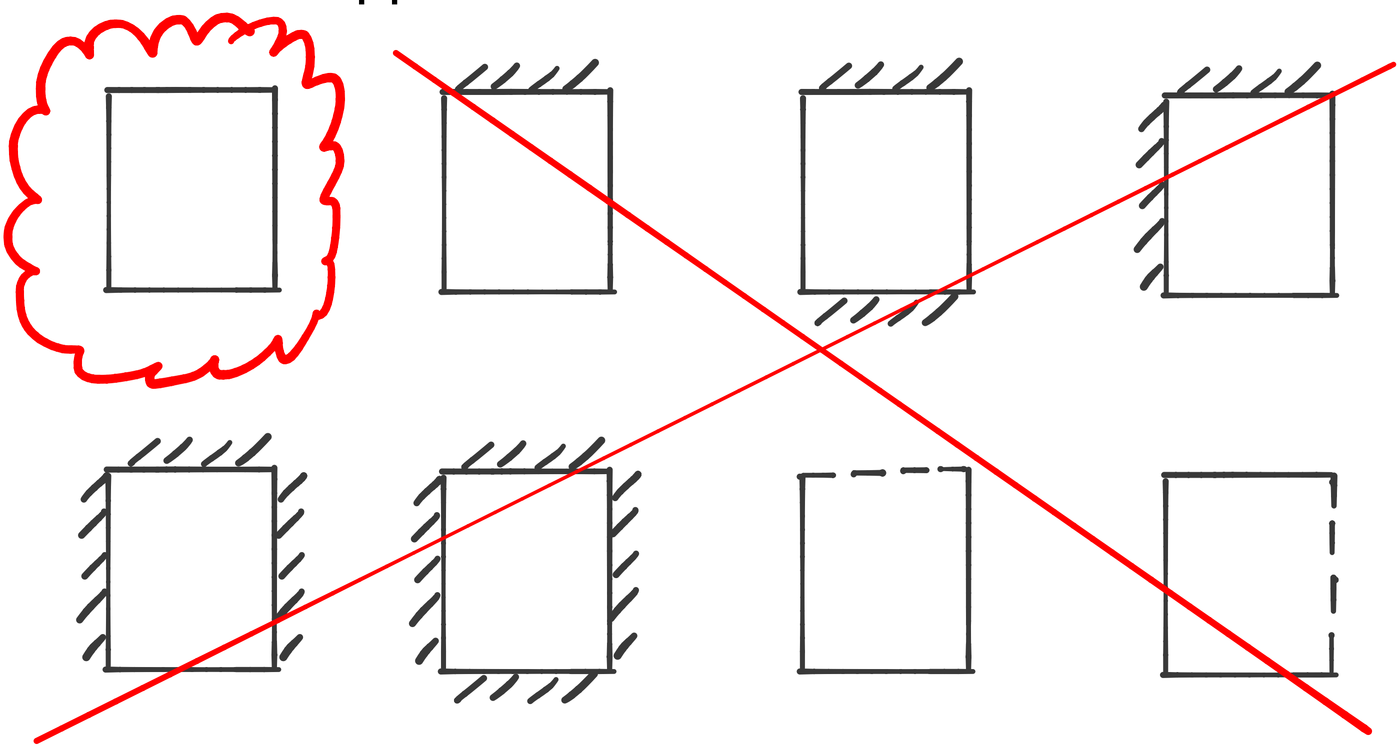


Czerny Tabel 2.2.1

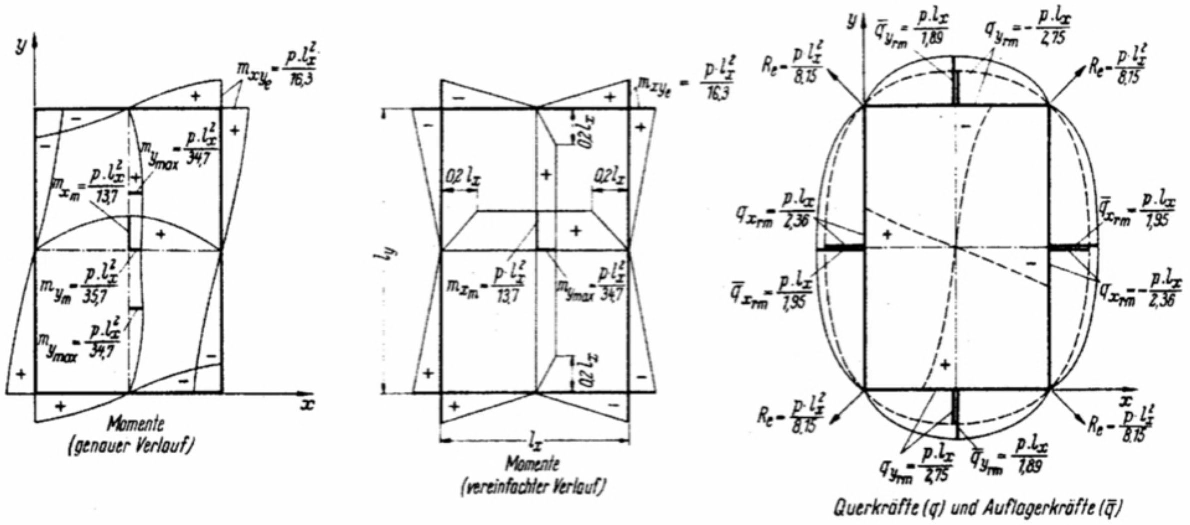
evenly loaded, foursided, free rotating support

2.2 Tafeln für gleichmäßig vollbelastete vierseitig gelagerte Rechteckplatten

2.2.1 Einspannungsfreie Lagerung der vier Ränder

$l_y : l_x$		1,00	1,05	1,10	1,15	1,20	1,25	1,30	1,35	1,40	1,45	1,50
$m_{xm} =$	$p \cdot l_x^2 :$	27,2	24,5	22,4	20,7	19,1	17,8	16,8	15,8	15,0	14,3	13,7
$m_{y\max} =$	$p \cdot l_x^2 :$	27,2	27,5	27,9	28,4	29,1	29,9	30,9	31,8	32,8	33,8	34,7
$m_{xye} = \pm$	$p \cdot l_x^2 :$	21,6	20,6	19,7	19,0	18,4	17,9	17,5	17,1	16,8	16,5	16,3
$R_e =$	$p \cdot l_x^2 :$	10,8	10,3	9,85	9,5	9,2	8,95	8,75	8,55	8,4	8,25	8,15
$q_{xrm} = \pm$	$p \cdot l_x :$	2,96	2,87	2,78	2,71	2,64	2,58	2,52	2,47	2,43	2,39	2,36
$\bar{q}_{xrm} =$	$p \cdot l_x :$	2,19	2,15	2,11	2,07	2,04	2,02	2,00	1,98	1,97	1,96	1,95
$q_{yrm} = \pm$	$p \cdot l_x :$	2,96	2,92	2,89	2,86	2,84	2,82	2,80	2,78	2,76	2,75	2,75
$\bar{q}_{yrm} =$	$p \cdot l_x :$	2,19	2,14	2,09	2,05	2,02	1,99	1,96	1,94	1,92	1,90	1,89
$f_m =$	$\frac{p \cdot l_x^4}{E \cdot d^3} \cdot$	0,0487	0,0536	0,0584	0,0631	0,0678	0,0728	0,0767	0,0809	0,0850	0,0890	0,0927

$l_y : l_x$		1,50	1,55	1,60	1,65	1,70	1,75	1,80	1,85	1,90	1,95	2,00
$m_{xm} =$	$p \cdot l_x^2 :$	13,7	13,2	12,7	12,3	11,9	11,5	11,3	11,0	10,8	10,6	10,4
$m_{y\max} =$	$p \cdot l_x^2 :$	34,7	35,4	36,1	36,7	37,3	37,9	38,5	38,9	39,4	39,8	40,3
$m_{xye} = \pm$	$p \cdot l_x^2 :$	16,3	16,1	15,9	15,7	15,6	15,5	15,4	15,3	15,3	15,2	15,1
$R_e =$	$p \cdot l_x^2 :$	8,15	8,05	7,95	7,85	7,8	7,75	7,7	7,65	7,65	7,6	7,55
$q_{xrm} = \pm$	$p \cdot l_x :$	2,36	2,33	2,30	2,27	2,25	2,23	2,21	2,19	2,18	2,16	2,15
$\bar{q}_{xrm} =$	$p \cdot l_x :$	1,95	1,94	1,93	1,92	1,92	1,92	1,92	1,92	1,92	1,92	1,92
$q_{yrm} = \pm$	$p \cdot l_x :$	2,75	2,74	2,73	2,73	2,73	2,72	2,72	2,71	2,71	2,70	2,70
$\bar{q}_{yrm} =$	$p \cdot l_x :$	1,89	1,88	1,87	1,86	1,85	1,84	1,83	1,82	1,82	1,82	1,82
$f_m =$	$\frac{p \cdot l_x^4}{E \cdot d^3} \cdot$	0,0927	0,0963	0,0997	0,1029	0,1060	0,1093	0,1118	0,1145	0,1169	0,1195	0,1215



Verlauf der Schnittgrößen für das Seitenverhältnis $l_y : l_x = 1,5$

Czerny Tabel 2.2.1

evenly loaded, foursided, free rotating support

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$m_{y\max} = p \cdot l_x^2 :$	27,2	27,5	27,9	28,4	29,1	29,9	30,9	31,8	32,8	33,8	34,7
$m_{xye} = \pm\, p \cdot l_x^2 :$	21,6	20,6	19,7	19,0	18,4	17,9	17,5	17,1	16,8	16,5	16,3
$R_e = p \cdot l_x^2 :$	10,8	10,3	9,85	9,5	9,2	8,95	8,75	8,55	8,4	8,25	8,15
$q_{xrm} = \pm\, p \cdot l_x :$	2,96	2,87	2,78	2,71	2,64	2,58	2,52	2,47	2,43	2,39	2,36
$\bar{q}_{xrm} = p \cdot l_x :$	2,19	2,15	2,11	2,07	2,04	2,02	2,00	1,98	1,97	1,96	1,95
$q_{yrm} = \pm\, p \cdot l_x :$	2,96	2,92	2,89	2,86	2,84	2,82	2,80	2,78	2,76	2,75	2,75
$\bar{q}_{yrm} = p \cdot l_x :$	2,19	2,14	2,09	2,05	2,02	1,99	1,96	1,94	1,92	1,90	1,89
$f_m = \frac{p \cdot l_x^4}{E \cdot d^3} \cdot$	0,0487	0,0536	0,0584	0,0631	0,0678	0,0728	0,0767	0,0809	0,0850	0,0890	0,0927

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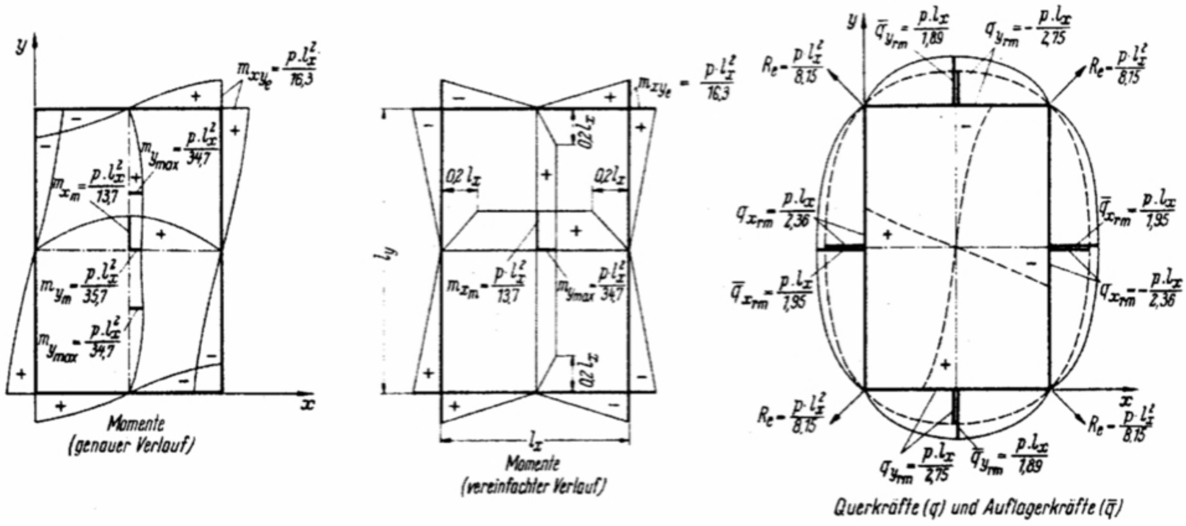
$m_{xm} = p \cdot l_x^2 :$	13,7	13,2	12,7	12,3	11,9	11,5	11,3	11,0	10,8	10,6	10,4
$m_{y\max} = p \cdot l_x^2 :$	34,7	35,4	36,1	36,7	37,3	37,9	38,5	38,9	39,4	39,8	40,3
$m_{xye} = \pm\, p \cdot l_x^2 :$	16,3	16,1	15,9	15,7	15,6	15,5	15,4	15,3	15,3	15,2	15,1
$R_e = p \cdot l_x^2 :$	8,15	8,05	7,95	7,85	7,8	7,75	7,7	7,65	7,65	7,6	7,55
$q_{xrm} = \pm\, p \cdot l_x :$	2,36	2,33	2,30	2,27	2,25	2,23	2,21	2,19	2,18	2,16	2,15
$\bar{q}_{xrm} = p \cdot l_x :$	1,95	1,94	1,93	1,92	1,92	1,92	1,92	1,92	1,92	1,92	1,92
$q_{yrm} = \pm\, p \cdot l_x :$	2,75	2,74	2,73	2,73	2,73	2,72	2,72	2,71	2,71	2,70	2,70
$\bar{q}_{yrm} = p \cdot l_x :$	1,89	1,88	1,87	1,86	1,85	1,84	1,83	1,82	1,82	1,82	1,82
$f_m = \frac{p \cdot l_x^4}{E \cdot d^3} \cdot$	0,0927	0,0963	0,0997	0,1029	0,1060	0,1093	0,1118	0,1145	0,1169	0,1195	0,1215



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m_{ymax} =	$p \cdot l_x^2 :$	27,2	27,5	27,9	28,4	29,1	29,9	30,9	31,8	32,8	33,8	34,7
m_{xye} = ±	$p \cdot l_x^2 :$	21,6	20,6	19,7	19,0	18,4	17,9	17,5	17,1	16,8	16,5	16,3
R_e =	$p \cdot l_x^2 :$	10,8	10,3	9,85	9,5	9,2	8,95	8,75	8,55	8,4	8,25	8,15
q_{xrm} = ±	$p \cdot l_x :$	2,96	2,87	2,78	2,71	2,64	2,58	2,52	2,47	2,43	2,39	2,36
$\bar{q}_{xrm}$ =	$p \cdot l_x :$	2,19	2,15	2,11	2,07	2,04	2,02	2,00	1,98	1,97	1,96	1,95
q_{yrm} = ±	$p \cdot l_x :$	2,96	2,92	2,89	2,86	2,84	2,82	2,80	2,78	2,76	2,75	2,75
$\bar{q}_{yrm}$ =	$p \cdot l_x :$	2,19	2,14	2,09	2,05	2,02	1,99	1,96	1,94	1,92	1,90	1,89
f_m =	$\frac{p \cdot l_x^4}{E \cdot d^3} \cdot$	0,0487	0,0536	0,0584	0,0631	0,0678	0,0728	0,0767	0,0809	0,0850	0,0890	0,0927

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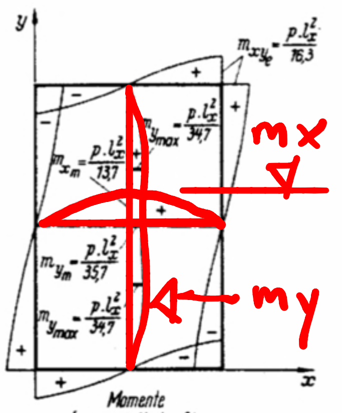
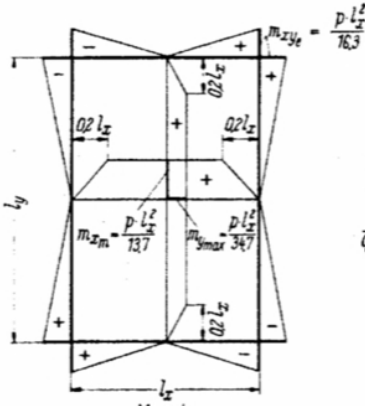
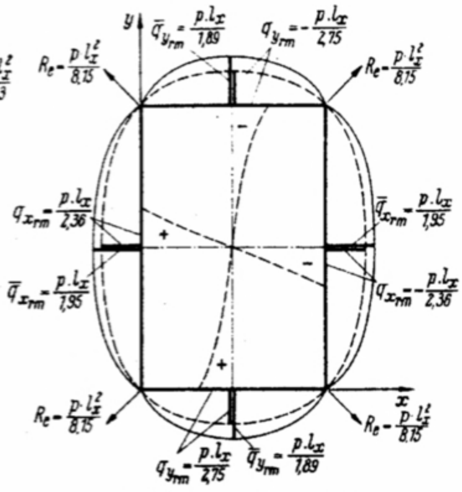
m_{xm} =	$p \cdot l_x^2 :$	13,7	13,2	12,7	12,3	11,9	11,5	11,3	11,0	10,8	10,6	10,4
m_{ymax} =	$p \cdot l_x^2 :$	34,7	35,4	36,1	36,7	37,3	37,9	38,5	38,9	39,4	39,8	40,3
m_{xye} = ±	$p \cdot l_x^2 :$	16,3	16,1	15,9	15,7	15,6	15,5	15,4	15,3	15,3	15,2	15,1
R_e =	$p \cdot l_x^2 :$	8,15	8,05	7,95	7,85	7,8	7,75	7,7	7,65	7,65	7,6	7,55
q_{xrm} = ±	$p \cdot l_x :$	2,36	2,33	2,30	2,27	2,25	2,23	2,21	2,19	2,18	2,16	2,15
$\bar{q}_{xrm}$ =	$p \cdot l_x :$	1,95	1,94	1,93	1,92	1,92	1,92	1,92	1,92	1,92	1,92	1,92
q_{yrm} = ±	$p \cdot l_x :$	2,75	2,74	2,73	2,73	2,73	2,72	2,72	2,71	2,71	2,70	2,70
$\bar{q}_{yrm}$ =	$p \cdot l_x :$	1,89	1,88	1,87	1,86	1,85	1,84	1,83	1,82	1,82	1,82	1,82
f_m =	$\frac{p \cdot l_x^4}{E \cdot d^3} \cdot$	0,0927	0,0963	0,0997	0,1029	0,1060	0,1093	0,1118	0,1145	0,1169	0,1195	0,1215



Momente (genauer Verlauf)



Momente (vereinfachter Verlauf)



Querkräfte (q) und Auflagerkräfte ($\bar{q}$)

Verlauf der Schnittgrößen für das Seitenverhältnis $l_y : l_x = 1,5$

4. Deformation

$$f_{el} = \frac{p_{d,ser} * l_x^{\,4}}{E * h^{3}} * Czerny$$

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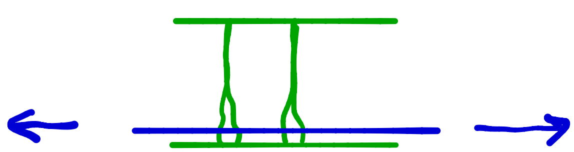
$$f_{el} = \frac{p_{d,ser} * l_x^{\ 4}}{E * h^{\ 3}} * \text{Czerny}$$

$6{,}7 \quad 625 \cdot 10^{12}$

$4'096'000$

$$f_{el} = \frac{6{,}7 \cdot 10^{-3} \cdot 5'000^4}{32'500 \cdot 160^3} \cdot 0{,}0327 = 2{,}92 \text{ mm}$$

↳ elastic, uncracked (homogeneous)



cracked × 3...5

$\underline{f_{eff}} \approx 4 \cdot 2{,}92 = \underline{11{,}7 \text{ mm}}$

$$\underline{f_{adm}} \approx \frac{l_x}{250 \ldots 350} \approx \frac{5000}{300} = \underline{16{,}7 \text{ mm}}$$

$\rightarrow f_{eff} < f_{adm} \rightarrow$ ☺

4. Deformation

$$f_{el} = \frac{p_{d,ser} * l_x^{\,4}}{E * h^{\,3}} * \text{Czerny}$$

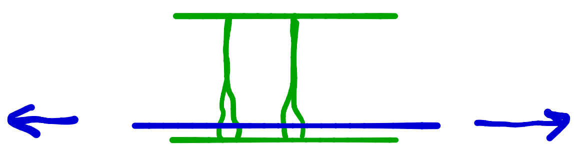
simple beam:

$$f_{el} = \frac{5 \cdot 6{,}7 \cdot 5'000^4}{384 \cdot 32'500 \, \frac{1000 \cdot 160^3}{12}}$$

$$= 3{,}7 \text{ mm}$$

$$f_{el} = \frac{6{,}7 \cdot 10^{-3} \cdot 5'000^4}{32'500 \cdot 160^3} \cdot 0{,}0927 = 2{,}92 \text{ mm}$$

↳ elastic, uncracked (homogeneous)



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$\rightarrow$ $f_{eff} < f_{adm}$ $\rightarrow$ ☺

6. Reinforcement

6a. Bending Moments

$$\boxed{m_{x/y} = \frac{p * l_x^{\,2}}{Czerny_{x/y}}}$$

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6a. Bending Moments

$$m_{x/y} = \frac{p * l_x^{2}}{Czerny_{x/y}}$$

always x

according direction

6. Reinforcement

6a. Bending Moments

$$m_{x/y} = \frac{p * l_x^{\,2}}{\text{Czerny}_{x/y}}$$

$$m_{dx} = \frac{12{,}3 \cdot 5{,}0^2}{13{,}7} = 22{,}4 \text{ kNm/m}'$$

$$m_{dy} = \frac{12{,}3 \cdot 5{,}0^2}{34{,}7} = 8{,}9 \text{ kNm/m}'$$

6. Reinforcement

6a. Bending Moments

$$m_{x/y} = \frac{p * l_x^{\,2}}{\text{Czerny}_{x/y}}$$

$$m_{dx} = \frac{12{,}3 \cdot 5{,}0^2}{13{,}7} = 22{,}4 \ kNm/m'$$

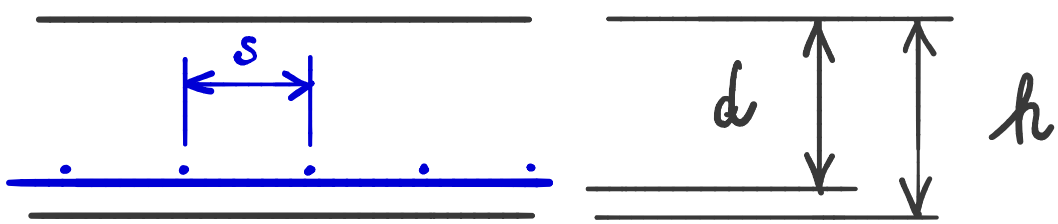
$$m_{dy} = \frac{12{,}3 \cdot 5{,}0^2}{34{,}7} = 8{,}9 \ kNm/m'$$

simple beam: $\frac{12{,}3 \cdot 5{,}0^2}{8} = 38{,}4$

6. Reinforcement

6b. Minimum Reinforcement As,min

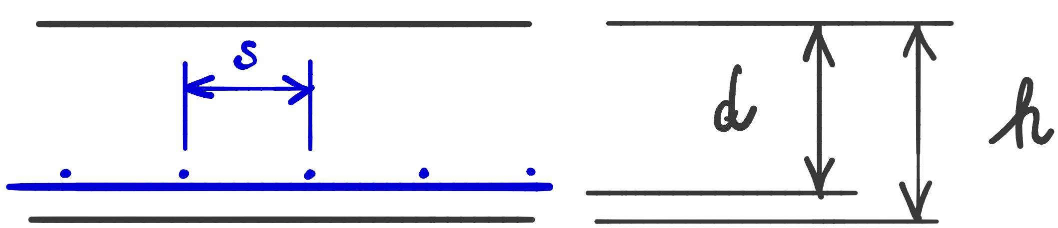
$$A_{s,min} = 0{,}15\% * d * 1'000$$



6. Reinforcement

6b. Minimum Reinforcement As,min

$$A_{s,min} = 0{,}15\% * d * 1'000$$



$$\underline{A_{s,min}} = 0{,}15\% \cdot 1000 \cdot 160 = \underline{240\ mm^2/m'}$$

→ e.g. ø 8 s = 200

6. Reinforcement

6c. Required Reinforcement $A_{s,req}$

$$A_{s,req} = \frac{m_d * 10^6}{0,9 * d * f_{dy}}$$

6. Reinforcement

6c. Required Reinforcement As,req

$$A_{s,req} = \frac{m_d * 10^6}{0{,}9 * d * f_{dy}}$$

$$A_{s,req} = \frac{22{,}4 \cdot 10^6}{0{,}9 \cdot 125 \cdot 435} = 458 \ mm^2/m$$

$> A_{s,min}$

$\rightarrow$ $\underline{\phi 10 \ \ s = 150}$

6. Reinforcement

6c. Required Reinforcement As,req

$$A_{s,req} = \frac{m_d * 10^6}{0,9 * d * f_{dy}}$$

$$A_{x,req} = \frac{22,4 \cdot 10^6}{0,9 \cdot 125 \cdot 435} = 458 \ mm^2/m$$

$> A_{s,min} = 240$

$\rightarrow$ $\underline{\phi 10 \quad s = 150}$

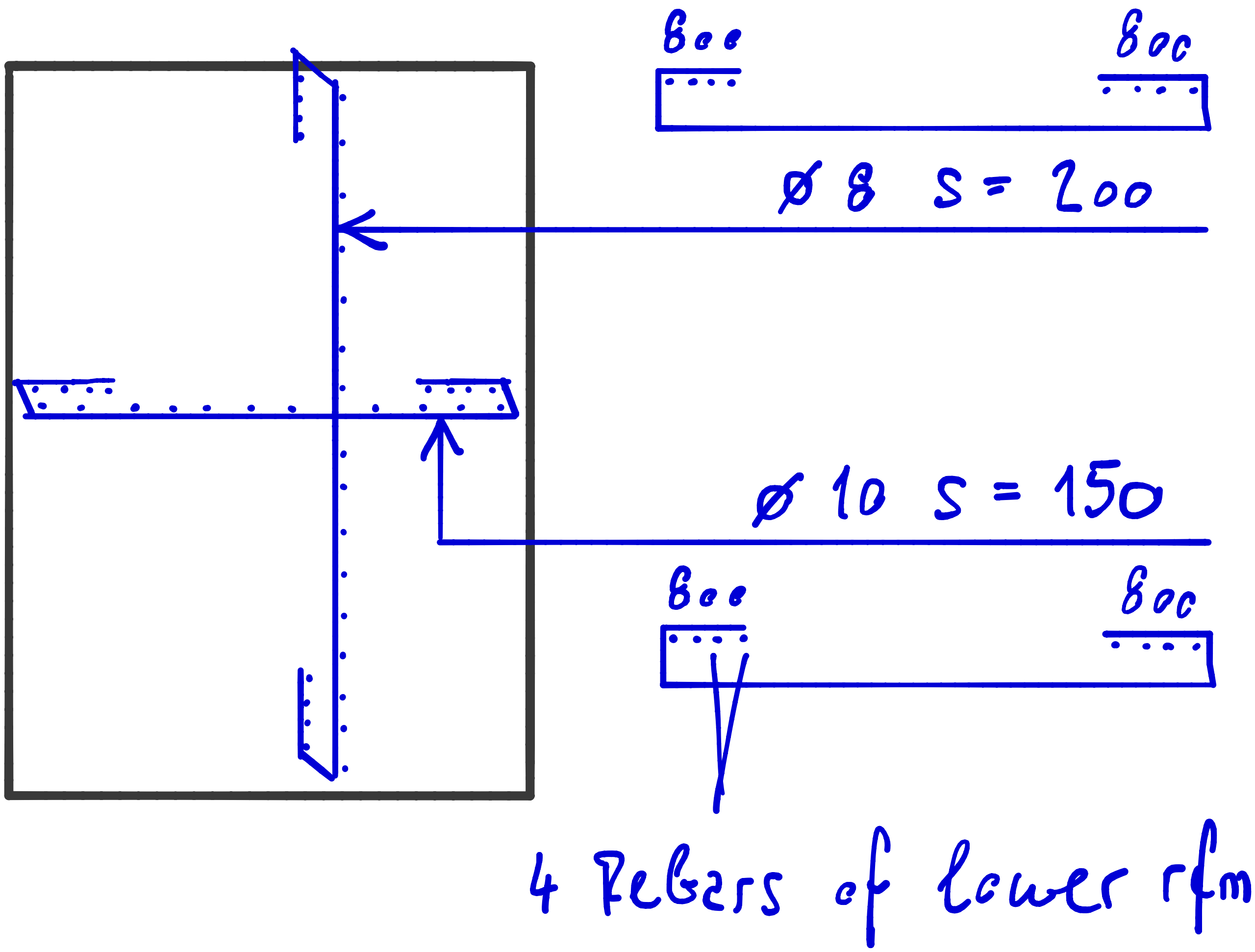
$$A_{y,req} = \frac{8,9 \cdot 10^6}{0,9 \cdot 125 \cdot 435} = 182 \ mm^2/m$$

$< A_{s,min} = 240$

$\rightarrow$ $\underline{\phi 8 \quad s = 200}$

6. Reinforcement

6d. Design of Reinforcement



blue → lower rfm

red → upper rfm (not necessary)

→ optional ø 8/200 (min.)

Summary

1. Dimensions $\ell_x \quad \ell_y \quad h \quad d$

2. Materials $E \quad f_{yd}$

3. Characteristic Loads $g \quad q \quad n$

4. Design Loads $P_{d,ser} \quad P_d$

5. Calculation of Deformation

f_{eff}

6. Design of Reinforcement

$A_{s\,min}, A_{sx}, A_{sy}$